Forecasting Realized Volatility of BIST Indices with Har-Type Models

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Abstract

In this paper, realized volatility of a selection of BIST Indices are forecasted with Heterogeneous Autoregressive Model (HAR) and its variations. For this purpose, ticks between 2001 and 2021 are used to generate 5-minute returns, which formed the basis for calculations of realized volatility and other realized measures. In the study, rolling windows are utilized for forecasting the volatility of one day ahead. These predictions are then compared to the actual realized volatilities. The study provides a thorough comparison of HAR-type models, and emphasizes the importance of underlying time series' characteristics in forecasting. Moreover, the findings of this paper also hint at matters of diversification particular to index volatility forecasting. In overall, HAR Models proved to be a successful estimator for Turkish Stock Exchange time series.

Keywords: Bist; har models; high frequency data; realized volatility; volatility forecasting

INTRODUCTION

The term volatility is an essential feature of finance. It denotes a financial security's degree of deviation from its measured, expected levels of return. Investment classes, therefore, are naturally expected to move up or down in price, while the unexpected rates of change at price levels have a say for the investors from all the complexity spectrum ranging from individual investors to institutions such as hedge funds. To this end, forecasting future volatility is crucial as it may help the decision-making process in portfolio allocation, assessing the risk versus return of holdings, and providing an anchor for various essential methods in financial risk management and derivatives pricing.

A recent trending approach to the forecasting of volatility is the calculation of realized variance by using high-frequency data. With the eventual evolution of computing performances to very high speed, the realized variance applications have made it into the spotlight and become the focus of attention for their capabilities in capturing and forecasting this deviation with exceptional precision.

This paper uses high-frequency data for Turkish Stock Exchange to forecast future volatility. Variations of the Heterogeneous Autoregressive Model (HAR) are used to forecast the forward realized variance of the major indices of Borsa Istanbul; BIST 100 (XU100), BIST 30 (XU030), BIST Financials (XUMAL), BIST Industrials (XUSIN), BIST Banks (XBANK), BIST Chem. Petrol Plastic (XKMYA), BIST Food Beverage (XGIDA), BIST Basic Metal (XMANA), BIST Metal Products Mach. (XMESY), BIST Nonmetal Min. Product (XTAST), and BIST Textile Leather (XTEKS). The research covered 21-year data between 2001 and 2021, from which the realized volatilities of 5-minute prices of those indices regressed per chosen models with a rolling window approach. The paper provides a thorough comparison of HAR-type models, and emphasizes the importance of underlying data's characteristics in forecasting. Moreover, the findings of this paper also hint at matters of diversification particular to index volatility forecasting. Study also suggests that using HAR-type models demonstrate successful performance as an estimator of volatility according to a selection of test metrics.

Related Literature

The methodology of financial modelling of volatility forecasting has taken off by the ARCH Model (Engle, 1982) and its later extension GARCH(Bollerslev, 1986).

In the literature, various GARCH-type models were incorporated with different probability distributions (Bali et al, 2008; Bali &Theodossiou 2007; Giot, 2003; Giot& Laurent 2003). Moreover, there are studies (Angelidis et al, 2004; Braione&Scholtes, 2016; Giot& Laurent, 2003; Kuester et al, 2006) which also concluded that the choice of volatility model is less relevant than the choice of the probability distribution.

Volatility forecasting was also approached as a stochastic problem rather than a deterministic one, as GARCH or ARCH suggests (Heston, 1993).

Volatility, in its essence, is a proxy of market anxiety, an immeasurable quantity. Price, unlike volatility, can be seen throughout a market session in a nearly continuous manner as transaction speed converges to split seconds. Drawing parallels from the price process to the problem of volatility forecasting, high-frequency data of price returns are utilized in a non-parametric framework. The volatility derived from this application was named as the realized volatility (RV). The valuable nature of the high-frequency return data first drew the attention in 1980s(Merton, 1980) as an inference from a hypothetical approach arguing for the instrumentality of the sum of squared high-frequency returns in forecasting conditional variance. Throughout that decade, high-frequency data has not seen interest possibly due to computing constraints in real-world practice. However, the last years of the millennium have seen increasing numbers of research papers in the area (Andersen, Bollerslev, Diebold,&Labys2001;Schwert, 1998; Taylor & Xu, 1997). The usage of high-frequency data for realized volatility is not confined to its suitability in forecasting, as there are researches underlining out its abilities in standardizing returns and its co-movement with correlation(Andersen, Bollerslev, Diebold, &Labys,2000;Andersen, Bollerslev, Diebold, &Ebens, 2001).

Starting with the new millennium, RV has begun to be the subject of the focusing glass to be covered in depth in terms of pros and cons with different implementations (Andersen et al, 2003; Barndorf-Nielsen & Shephard 2002; Giot& Laurent, 2004; Martens et al, 2009).

The HAR-RV Model(Corsi, 2009) has had the most credit in modelling the RV for prediction purposes. The term heterogeneity is derived from the heterogeneous market hypothesis (Müller et al,1993), which refers to the heterogeneity of the market actors with different trading motives and time preferences. For example, day traders are more accountable for intra-day volatility. In contrast, position traders move their positions with weekly adjustments, and institutional investors act upon longer time horizons for their investments, thus creating volatility in different levels. Grounding on this intuition, HAR-RV model parameterized daily, weekly, and monthly aggregated realized volatilities to forecast the one-day ahead volatility and compared this model against simple autoregressive models and ARFIMA, marking its success in predicting future volatility and capturing long-memory characteristics as good as its counterparts, while being effort-efficient.

Following this first step, the HAR model has seen variations to cover various real sample problems. For instance, HAR-RV-CJ and HAR-RV-J models (Andersen et al, 2007) where the daily RV defined in two parts; the continuous path of price volatility and spikes in that path named jumps. The first model considers the contribution of jumps and continuous paths distinctly whereas as the latter treats only jumps separately. Another extension proposed to the main model wasa leverage component (Leverage HAR model or LHAR),which included negative returns as a distinct, independent variable in the model for future volatility (Corsi and Reno, 2012). Finally,there were also trials to separate the integrated variance and error term in the realized variance formulation with the proposal of the HARQ model(with also variations as CHARQ and HARQ-Jmodel), pointing out the conditionality of the variance of the error term (Bollerslev et al, 2016). Quarticity, as those errors were named, lies in the course of realized variance naturally and has a changing density in the process, in which sometimes it relieves and gives way to realized variance to forecast this integrated variance efficiently, and when in times it becomes noisier and makes the forecasting inaccurate with RV.

The literature on forecasting volatility dependent on realized measures and the HAR model has tested its validity onvariety of financial time series such as indices(Chen et al, 2018; Kambouroudis et al, 2021;Todorova &Soucek 2014; Wang, 2009),foreign exchange rates

(AgermarkandHoti, 2016), andbonds (Özbekler et al, 2021). There are also studies that review commodities and HAR Models together such as on energy commodities(Prokopczuk et al, 2016;Tang et al, 2021) or agricultural commodities (Degiannakis et al, 2022; Luo et al, 2022).The HAR model has also been tested on cryptocurrencies. (Bergsli et al, 2022;Ftiti et al, 2021)

There are few studies on Turkish high-frequency data using HAR Models for volatility modelling and forecasting on BIST 30(Çelik andErgin, 2014) and BIST 100 (Eroğlu et al, 2021;Türensal, 2021)

Data

The data used in this paper spans 5,103 working days of the Borsa Istanbul Stock Exchange between 02.01.2001 and 30.04.2021, with inclusive boundaries. For this period, high-frequency data are obtained from BIST DataStore (datastore.borsaistanbul.com) for the XU100, XU030, XUMAL, XUSIN, XBANK, XKMYA, XGIDA, XMANA, XMESY, XTAST, and XTEKS indices. The reasoning behind the selection among the whole list of BIST indices depends on the constituent number and the market capitalization of the indices.The selection is also pertinent to coverage of firm-level idiosyncrasies and providing both diversified and concentrated analysis of the volatility process, as relevant literature suggests (Campbell et al, 2001). The intraday values for those indices are then sampled into 5-minute returns for continuous trading sessions in a day, thus resulting in 378,560 observations. The formula for the derivation of the return series is the same as in equation (2), and for the calculation of RV, equation (4) is applied. The log and square root transformations of the RVs are not different from the generic mathematical technique.

The time space between price observations for return calculations is in line with the findingsin the literature (Liu et al, 2015). For forecasting methodology, the study utilizes 274 days of rolling windows for predicting the one-day ahead volatility, equating to 4,829 daily forecasts. The parsing of the data and forecasting are done through R and its package “highfrequency”. The performance evaluation sequence is conducted with R and package “MCS”.

In this study, days with two trading sessions were combined into one whole trading session in resemblance to the current state. The unchanging values between two trading sessions are also omitted from that day's array. The values of the closing auctions are also included in the analysis. No virtual data generation was needed due to the availability of sufficient frequencies.

The generation of descriptive statistics of the data is operated on the 3 RV series (RV, log RV, square root RV) per each of the indices and with a rolling window basis. To put it differently, no whole series for an index is adopted for the computation of these statistics; every 274-day window is treated separately to provide a series of values for statistics (Mean of the means, mean of the maximums etc.). This approach is necessary to treat each window as a separate sample space, in total alignment with the adopted forecasting procedure. The tables for descriptive statistics of all series are displayed below (see tables 1 to 3).

To review and make a general assessment of all these statistics, the most critical comment that needs to be remarked on is the log series' closeness to normal distribution standards. Although not every p value for Jarque-Bera statistics of log series is above usual confidence levels of %1 and %5, the significant proximity of Kurtosis levels to 3 and skew levels to zero in comparison to other counterparts mark its usefulness in these types of analyses. In addition, the plain RV series' underperformance in contrast to its transformed peers should also be another aspect that needs to be paid attention to. Briefly, the log transformation of RV series proves to be an appropriate input for volatility forecasting purposes due to its convergence to normal distribution.

Methods

Realized Volatility (RV)

The price process of a sample financial asset P_t can be determined by the stochastic differential equation:

$$dP_t = \mu_t d_t + \sigma_t \, dW_t, \tag{1}$$

Where μ_t and σ_t denote the drift and the instantaneous volatility processes, respectively, and W_t is a standard Brownian motion. From this formulation, the return for the i-th interval of a trading day can be defined as:

$$r_{t,i} = P_{t-1+i\Delta}/P_{t-1+(i-1)\Delta} - 1, \tag{2}$$

$$i = 1,2, \dots, M,$$

Where M equals $1/\Delta$ to denote the sampling frequency. As this frequency goes to infinity, the formulation will start to cover the infinitesimal price movements, enabling to formulate the integrated volatility for a given day as:

$$IV_t = \int_{t-1}^{t} \sigma_s^2 \, \mathrm{d}s. \tag{3}$$

However, this application is impossible in normal conditions due to the discrete nature of the price process and market anomalies that appear as this process converges to continuous state. Instead, realized volatility RV formulation replaces to become a depiction of integrated volatility as:

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2 \tag{4}$$

TheRV_there is the summation of the square of intraday returns for a day$t$. As$M \to \infty$, the number of return calculations within a day creates enough samples to replicate and give an idea of the integrated volatility for a given day. As mentioned earlier, it is important to limit the number of return instances to a reasonable degree in order to provide a sound analysis that is not diluted with market inefficiencies. For that reason, this study involves a 5-minute intra-day returns in measurement of the realized volatility.

Forecasting Methodology

HAR Model (HAR-RV)

Corsi (2009) designed HAR-RV model as a cascade of daily, weekly, and monthly RVs to forecast the one day ahead volatility:

$$RV_{t+1}^{d} = c + \beta^{d} RV_{t}^{d} + \beta^{w} RV_{t}^{w} + \beta^{m} RV_{t}^{m} + \varepsilon_{t+1}^{d}, \tag{5}$$

whereV_t^d , RV_t^w and RV_t^m represent the daily, weekly and monthly realized volatility. The daily RV is calculated as noted in the equation (4), whereas weekly and monthly realized volatility are the average of the past values of daily RVs according to the size of the frequency. The model is estimated with an ordinary least squares method.

HAR Model with Jumps (HAR-RV-J)

Andersen et al. (2007) separated the daily RV into two different parts; the continuous path of price volatility and spikes in that path named jumps. To consider and define the effects of jumps to future volatility, they proposed two HAR model variations, specified as HAR-RV-J and HAR-RV-CJ. The first model they proposed was HAR-RV-J, where:

$$RV_{t+1}^{d} = c + \beta^{d} RV_{t}^{d} + \beta^{w} RV_{t}^{w} + \beta^{m} RV_{t}^{m} + \beta^{jd} {J_{t}}^{d} + \varepsilon_{t+1}^{d}, \tag{6}$$

In addition, they defined Jumps (J_t) as:

$$J_t = max[RV_t - BPV_t; 0], \tag{7}$$

where RV_t stands for the realized volatility, and BPV_t stands for the bipower variation. Bipower variation (Barndorff-Nielsen and Shephard, 2004)) is defined as:

$$BPV_t \ = \ \mu_1^{-2} \sum_{i=k+1}^{M} |r_{t,i}||r_{t,i-k}|, \tag{8}$$

$$k \geq 0,$$

where $\mu_1 = \sqrt{\frac{2}{\pi}} = E(Z)$ is a representation of the mean of the absolute value of the Standard Gaussian random variable.

HAR Model with Quarticity and Jumps (HARQJ)

In volatility literature, RV is viewed as a proxy of the latent integrated volatility (IV) process. Therefore, existence of measurement errors in RV causes attenuation bias in forecasting with HAR modelling of the volatility, particularly when the variances of measurement errors are high. To address this variability in error variance, Bollerslev et al. (2016) proposed to modify the daily realized volatility partition of the fundamental HAR model with quarticity and also included jumps, same as in the HAR-RV-J model of Andersen et al. (2007) that is mentioned previously:

$$RV_{t+1}^{d} \ = \ c \ + \ \left(\beta^{d} + \beta_{Q}^{d} RQ_{t}^{1/2}\right) RV_{t}^{d} \ + \ \beta^{w} RV_{t}^{w} + \beta^{m} RV_{t}^{m} \ + \beta^{jd} {J_{t}}^{d} + \varepsilon_{t+1}^{d} \tag{9}$$

Note that the equation in the first parenthesis takes place of the coefficient of the daily RV, and RQ_t denotes the Realized Quarticity, which is formulated as:

$$RQ_t = \frac{M}{3}\sum_{i=1}^{M} r_{t,i}^4. \tag{10}$$

Continuous HAR Model (CHAR)

Grounding on the findings of the Andersen et al. (2007), Bollerslev et al. (2016) also defined the Continuous HAR Model (CHAR) as:

$$RV_{t+1}^d = c + \beta^d BPV_t^d + \beta^w BPV_t^w + \beta^m BPV_t^m + \varepsilon_{t+1}^d, \tag{11}$$

Continuous HAR Model with Quarticity (CHARQ)

Bollerslev et al. (2016) also combined their CHAR model with quarticity by formulation:

$$RV_{t+1}^d = c + \left(\beta^d + \beta_Q^d TPQ_t^{1/2}\right)BPV_t^d + \beta^w BPV_t^w + \beta^m BPV_t^m + \varepsilon_{t+1}^d, \tag{12}$$

in which BPV notation for daily, weekly, and monthly estimators are identical to the CHAR model. However, the model differentiates from HARQJ model in terms of its treatment towards quarticity. To constitute a jump robust estimator of quarticity, model uses Tripower Quarticity (TPQ) of Barndorff-Nielsen and Shephard (2006):

$$TPQ_t \equiv M\mu_{4/3}^{-3}\sum_{i=1}^{M-2}\left|r_{t,i}\right|^{4/3}\left|r_{t+1,i}\right|^{4/3}\left|r_{t+2,i}\right|^{4/3}, \tag{13}$$

where$\mu_{4/3} \equiv 2^{2/3}\Gamma(7/6)/\Gamma(1/2) = E\left(|Z|^{4/3}\right)$.

Performance Evaluation Methodology

For performance comparison for forecasting methodologies, the Model Confidence Set (MCS) (Hansen et al, 2011) is employed. In its basis, the model aims to find the best fitting model or a selection of best fitting models by executing an iteration of loss functions until to the point where models that underperform are omitted and models that provide eligible results in predetermined confidence interval survives. Calculated values of loss functions of a certain model are averaged then compared against average value for all models in that sample. It should also be noted that the initial set of models can stay as the best performing models with no worse performing model.

In this study, the 6 loss functions occupied in the MCS process are:

1. Squared Error 1: $SE_{1,t} = (\tilde{\sigma}_t - \hat{\sigma}_t)^2$ (14)

2. Squared Error 2: $SE_{2,t} = \left({\tilde{\sigma}_t}^2 - {\hat{\sigma}_t}^2\right)^2$ (15)

3. QLIKE Loss Function: $QLIKE_t = log\left({\hat{\sigma}_t}^2\right) + {\tilde{\sigma}_t}^2{\hat{\sigma}_t}^{-2}$ (16)

4. R^2 Log Loss Function: $R^2LOG_t = [log\left({\tilde{\sigma}_t}^2{\hat{\sigma}_t}^{-2}\right)]^2$ (17)

5. Absolute Error 1: $AE_{1,t} = |\tilde{\sigma}_t - \hat{\sigma}_t|$ (18)

6. Absolute Error 2: $AE_{2,t} = |\tilde{\sigma}_t{}^2 - \hat{\sigma}_t{}^2|$ (19)

Figures with ~ denote the actual observations, whereas ^ denote the forecasted values.

RESULTS

The results of the forecasting procedure will be presented in this section, along with guiding commentary and decisive remarks for findings. The evaluative process for models is indicated in previous section as Model Confidence Set (MCS). The outputs of the MCS are tabulated for every index, thus adding up to 11 separate tables (see table 5 to 15). In addition, a table for combined results (see table 4) is also constituted to exhibit a general outlook. Finally, a series of figures for forecasts are given (see figure 1 to 5). In overall, the models have displayed a satisfying performance in MCS tests.

In index tables, the score of every loss function calculated are given and the model's ranking depending on those values are depicted. The average ranking for a model is calculated as the averages of these rankings, and in tests where a model is eliminated, the ranking is interpreted as 15. Cells that include "E" in tables denote the model's elimination in the loss function test. For general results, a general average ranking that shows a model's average ranking across index series, as well as minimums, maximums, and standard deviations of these rankings, are displayed. Counts of elimination from tests are also given.

In the combined results table, top half is mainly consisted of models with transformations,with log models taking the lead for average ranking among all series.Models which were run on plain RV series, however, largely form the last rows of general rankings.

The plain HAR-RV, HAR-RV-J, and CHAR models with log series retained the first rankings in the respective order, proving themselves as the best models.

In general, models with quarticity underperformed their non-quarticity competitors. While in overall this is the case, there are also index series where models with quarticity performed significantly better (XKMYA and XGIDA). In these indices, separate modelling of jumps also produced similar outcomes, though models with jumps also acquired good results in differing indices.

Forecast results are given with a collection of figures (see figure 1 to 5) for plain HAR-RV model on log series. Although being short in magnitude, the forecasts follow the actual values even in tail events,as can be seen from the graphs.

DISCUSSION

This paper aimed to provide a comparison of the HAR Model extensions for the purpose of forecasting volatility for BIST indices. As stated in the founding paper of Corsi (2009), the model aligns with the nature of high-frequency data through modelling and averaging the previous RV values, making the long memory count. The study spans 5-minute returns between 2001 and 2021. For forecasting, approximately one-year rolling windows are used. The predictions of the models are compared to the actual realized volatilities.

The first topic to discussin this research is the underlying time series properties. From descriptive statistics, the transformed series' convergence to normal distribution standards acquired as a fact, with logarithmic transformation stands one step further. This finding is also supported in comparison section, while models with transformations dominated the general statistics, particularly log models.The plain RV series on the other hand do not emerge as suitable for forecasting RV depending on these findings since they largely form the last rows of general rankings. In brief, the benefits of using logarithmic transformations are clear.

Another point that can be made out of this study is related with the plain HAR-RV model's success among all models. While run on log series, the basic model stands out as a robust RV estimator. This result suggests that there may be no need for extensions, differing treatments for jumps and attenuation biasesfor a robust RV examination. Although the paper's findings demand a remark on the plain model's performance, it would be wiser not to leave the models with extensions because of the reasons laid out in the upcoming paragraph.

The surfacing results on the underperformance of models with quarticity remains as a significant issue. While the tempting nature of commenting on this apparent overall underachievement is understandable, one should also take into account the typicalities of the underlying series. As noted previously in the results section, there are also index series where models with quarticity performed better. The unbalanced nature in terms of weights of these indices where quarticity models retained good rankings inclines the final judgement towards the usefulness of these models in this particular type of index time series. For a well-diversified portfolio, however, it is evident that daily realized volatility values should not be excluded from the RV forecasts even in the possible presence of noise contamination. A similar situation of successfulness in concentrated portfolios also holds for models with jumps, though it would be invalid to state a general comment in this sense about the models with jumps as they werealso successfulin diversified portfolios. Further research topic in this field could be on discerning the right conditions where quarticity and jump models differ in performances with paying regard to weights of the portfolio at hand.

As a final remark, HAR-type models proved to be a reliable estimator in index series. In overall, the models have displayed a satisfying performance in MCS tests. It is also apparent from the forecasting figures in which only outlying values are not fulfilling, where even in these, the direction and magnitude of the forecast hint at marginal values.

CONCLUSION

The problem of forecasting volatility is an important topic in modern finance. The venture starts with finding the proper definition: there is even dispute over the term volatility's description, and eventually the calculation methodology. The study at hand relies on the high-frequency expansion of the definition of volatility, where the quadratic variation equation finds parity between realized volatility and the ultimate goal: integrated volatility. As the whole quantitative finance literature on volatility suggests, volatility is, in its essence, an indecisiveness of the market while trying to find the right direction. Therefore, it is intuitive to look for the very incremental steps of this random walk.

By modelling RV in HAR models, our study concluded that underlying time series properties are an important determinant in forecasting of volatilities. In addition, plain

HAR Model proved itself as a sound way to start a forecasting process for RV. However, it is also evident thatmodel extensions should not be delisted from the forecasting tools, as they can come handy while there are issues related to diversification of the underlying indices. As a general conclusion, HAR models proved as a robust estimator throughout volatility forecasting processes in indices.

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TABLES

Table 1: Descriptive Statistics for Plain RV Series

	Mean	Min	Max	Median	Std	Skew	Kurtosis	JB	JB_p_value
XBANK	0.00056	0.00013	0.00499	0.00046	0.00045	4.72117	44.40573	49213.78369	0.00026
XGIDA	0.00053	0.00008	0.00491	0.00043	0.00046	4.92166	48.15518	59297.86088	0.00000
XKMYA	0.00048	0.00009	0.00394	0.00040	0.00039	4.84293	45.61071	46292.29030	0.00000
XMANA	0.00082	0.00016	0.00700	0.00070	0.00064	4.91936	52.86459	80858.53881	0.00993
XMESY	0.00037	0.00009	0.00387	0.00030	0.00034	5.44877	54.25260	70643.76410	0.00000
XTAST	0.00019	0.00003	0.00285	0.00013	0.00025	6.20532	61.13431	75381.79969	0.00000
XTEKS	0.00039	0.00008	0.00457	0.00029	0.00042	4.80619	42.26001	48998.24920	0.00000
XUMAL	0.00038	0.00008	0.00397	0.00029	0.00036	5.03754	46.74744	51568.29105	0.00001
XUSIN	0.00020	0.00004	0.00310	0.00014	0.00026	6.15020	60.66172	70591.47652	0.00000
XU030	0.00031	0.00006	0.00333	0.00023	0.00031	5.14404	47.41936	49443.14728	0.00000
XU100	0.00025	0.00005	0.00317	0.00019	0.00029	5.42863	50.68495	55303.49069	0.00000

Table 2: Descriptive Statistics for Log RV Series

	Mean	Min	Max	Median	Std	Skew	Kurtosis	JB	JB_p_value
XBANK	-7.92312	-9.43929	-5.69400	-7.95890	0.55133	0.56001	4.61433	78.08776	0.01957
XGIDA	-8.06785	-9.71458	-5.66029	-8.10775	0.62393	0.46219	4.36226	82.09340	0.10360
XKMYA	-8.18722	-9.71998	-5.85564	-8.22019	0.58892	0.54930	4.65080	101.72463	0.06390
XMANA	-7.53399	-9.00221	-5.37320	-7.56067	0.51413	0.55610	5.67388	207.22012	0.05601
XMESY	-8.38158	-9.75913	-5.93949	-8.43862	0.57475	0.80422	4.95900	117.81813	0.02186
XTAST	-9.01365	-10.43426	-6.23396	-9.10801	0.63005	1.10494	5.59179	196.50136	0.00120
XTEKS	-8.15818	-9.52453	-5.85852	-8.23405	0.56475	0.87855	4.93064	120.22107	0.00608
XUMAL	-8.32164	-9.80545	-5.95227	-8.37389	0.57922	0.70128	4.66592	82.92655	0.01413
XUSIN	-9.03578	-10.53858	-6.28293	-9.11478	0.62675	0.94926	5.18690	122.35768	0.00812
XU030	-8.51552	-10.06281	-6.08992	-8.57491	0.59293	0.73255	4.70994	84.52421	0.00576
XU100	-8.73252	-10.31706	-6.17810	-8.80123	0.62057	0.77928	4.72382	85.88539	0.00246

Table 3: Descriptive Statistics for Square Root RV Series

	Mean	Min	Max	Median	Std	Skew	Kurtosis	JB	JB_p_value
XBANK	0.02106	0.00999	0.06359	0.01991	0.00653	2.22747	14.66825	4020.93679	0.00601
XGIDA	0.02017	0.00835	0.06452	0.01892	0.00704	2.26512	16.17822	8229.25303	0.01382
XKMYA	0.01910	0.00854	0.05829	0.01805	0.00624	2.29585	15.27003	4357.37325	0.00630
XMANA	0.02559	0.01185	0.07572	0.02450	0.00755	2.51447	20.91595	12432.26633	0.01135
XMESY	0.01695	0.00842	0.05648	0.01582	0.00572	2.68362	18.73863	9150.53166	0.00000
XTAST	0.01224	0.00564	0.04834	0.01100	0.00505	3.20496	21.49277	8428.88014	0.00000
XTEKS	0.01807	0.00880	0.05921	0.01661	0.00632	2.52019	15.54608	4947.27535	0.00000
XUMAL	0.01728	0.00827	0.05631	0.01610	0.00585	2.44171	15.52336	4344.11242	0.00986
XUSIN	0.01222	0.00556	0.04896	0.01111	0.00490	3.02038	19.66731	5979.94616	0.00000
XU030	0.01563	0.00722	0.05230	0.01446	0.00551	2.49834	15.48702	3928.37840	0.01213
XU100	0.01409	0.00632	0.05042	0.01290	0.00534	2.64027	16.47770	4517.20227	0.00118

Table 4: Combined Results of MCS

		General Average Ranking	Minimum Average Ranking	Maximum Average Ranking	Standard Deviation of Average Rankings	Count of Elimination
1	**HAR_log**	6.20	4.17	8.83	1.20	9
2	**HARJ_log**	6.27	4.83	7.67	0.89	7
3	**CHAR_log**	6.47	4.67	12.33	2.55	14
4	**HARQJ_log**	7.68	3.00	9.50	2.00	7
5	**HARJ_sqrt**	8.09	7.00	9.67	0.76	24
6	**CHAR_sqrt**	8.33	7.17	12.33	1.38	23
7	**CHAR**	8.89	7.67	11.33	1.27	25
8	**HAR_sqrt**	9.30	7.50	12.33	1.62	25
9	**CHARQ_sqrt**	9.79	8.17	12.50	1.40	26
10	**CHARQ_log**	10.26	7.67	13.33	2.01	16
11	**CHARQ**	11.02	9.33	12.50	0.95	33
12	**HARQJ_sqrt**	11.44	8.00	14.00	1.40	24
13	**HAR**	11.97	9.50	14.00	1.19	42
14	**HARQJ**	12.61	10.67	14.17	0.96	34
15	**HARJ**	12.98	11.00	15.00	1.22	35

Table 5: Results of MCS for Forecasting RV of XBANK

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Average Ranking
CHAR_log	4	-1.0987	7	-0.9861	12	-0.8786	3	-0.1135	1	-1.0784	1	-2.3379	4.67
HAR_log	6	-1.0156	5	-1.0330	11	-0.9316	5	1.3698	2	-0.0539	2	-2.1337	5.17
HARJ_log	7	-0.9848	9	-0.6721	10	-0.9982	1	-1.4122	3	0.0904	4	-1.0130	5.67
CHAR_sqrt	2	-1.3473	3	-1.0983	5	-1.1489	E	E	E	E	3	-1.0713	7.17
HARJ_sqrt	1	-1.3684	2	-1.1455	8	-1.0841	E	E	E	E	6	0.2485	7.83
HARQJ_log	12	0.8885	13	1.0378	9	-1.0022	2	-0.3710	4	0.5768	9	0.5400	8.17
HAR_sqrt	5	-1.0299	4	-1.0969	3	-1.2240	E	E	E	E	8	0.4547	8.33
CHAR	8	-0.9579	1	-1.1568	1	-1.4324	E	E	E	E	10	1.6070	8.33
CHARQ_log	10	0.7654	15	1.2521	13	-0.8724	4	0.3779	5	0.7121	5	0.0908	8.67
CHARQ_sqrt	3	-1.3428	8	-0.9663	6	-1.1301	E	E	E	E	7	0.4177	9
CHARQ	9	-0.5396	6	-1.0038	2	-1.3081	E	E	E	E	E	E	10.33
HARQJ_sqrt	11	0.8580	12	1.0339	4	-1.1676	E	E	E	E	11	1.7233	11.33
HARQJ	14	1.5105	14	1.0871	7	-1.0962	E	E	E	E	E	E	13.33
HAR	15	1.5821	10	0.4267	14	0.6580	E	E	E	E	E	E	14
HARJ	13	1.3440	11	1.0172	15	1.0709	E	E	E	E	E	E	14

Table 6: Results of MCS for Forecasting RV of XGIDA

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
HARQJ_log	4	-1.5616	4	-0.8900	E	E	1	-1.5788	1	-1.5448	1	-2.8689	4.3
HARJ_log	3	-1.6825	5	-0.8785	E	E	E	E	2	0.1695	2	-2.3562	7
HARJ_sqrt	1	-3.4180	1	-1.6438	7	1.1105	E	E	E	E	8	1.6620	7.83
HARQJ_sqrt	2	-1.7659	6	-0.8054	5	-0.3268	E	E	E	E	5	1.1609	8
HAR_sqrt	5	-1.2634	2	-1.2196	6	0.0841	E	E	E	E	7	1.4306	8.33
HAR_log	6	-0.4483	11	0.6540	E	E	E	E	3	1.0639	3	-1.9494	8.83
HARQJ	9	0.4410	7	-0.1307	3	-2.0426	E	E	E	E	E	E	11
HARJ	10	0.6370	3	-1.0049	8	1.1276	E	E	E	E	E	E	11
HAR	12	1.1875	8	0.0203	2	-2.1908	E	E	E	E	E	E	11.17
CHAR	13	1.2179	9	0.1605	1	-2.3978	E	E	E	E	E	E	11.33
CHAR_log	11	0.8995	14	1.3871	E	E	E	E	E	E	4	0.4443	12.33
CHAR_sqrt	7	-0.3808	12	0.7468	10	1.3082	E	E	E	E	E	E	12.33
CHARQ	15	1.6975	10	0.4121	4	-1.2945	E	E	E	E	E	E	12
CHARQ_sqrt	8	0.4118	13	0.8663	9	1.2242	E	E	E	E	E	E	12.5
CHARQ_log	14	1.5535	15	1.7114	E	E	E	E	E	E	6	1.38753	13.33

Table 7: Results of MCS for Forecasting RV of XKMYA

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
HARQJ_log	1	-2.3910	4	-1.4043	10	-1.1325	1	-0.8349	1	-1.5004	1	-2.9924	3
HARJ_log	5	-1.2409	10	-0.6012	8	-1.1364	2	0.8349	3	0.6689	4	-0.9332	5.33
HAR_log	4	-1.7126	1	-1.5678	11	-1.1294	E	E	2	0.5583	2	-2.3646	5.83
HAR_sqrt	3	-1.8043	2	-1.4656	6	-1.1504	E	E	E	E	5	0.5962	7.67
CHAR_log	6	-1.1791	6	-1.2886	9	-1.1352	E	E	E	E	3	-1.1320	9
CHAR_sqrt	7	-0.7501	7	-1.0798	4	-1.1552	E	E	E	E	6	0.8294	9
HARJ_sqrt	2	-1.8272	3	-1.4305	5	-1.1513	E	E	E	E	E	E	9.17
HARQJ_sqrt	10	0.6173	12	0.9918	3	-1.1569	E	E	E	E	7	1.4364	10.33
CHAR	8	-0.1409	9	-0.8877	1	-1.1780	E	E	E	E	E	E	10.5
CHARQ_sqrt	11	0.9774	11	-0.2265	2	-1.1574	E	E	E	E	E	E	11.5
CHARQ_log	14	1.4163	14	1.1188	7	-1.1370	E	E	E	E	8	1.4605	12.17
HAR	12	1.0790	5	-1.3091	12	-1.0897	E	E	E	E	E	E	12.33
CHARQ	9	0.5750	8	-1.0297	13	-1.0847	E	E	E	E	E	E	12.5
HARQJ	13	1.1313	13	1.0076	14	0.8487	E	E	E	E	E	E	14.17
HARJ	15	1.4239	15	1.1647	15	1.0217	E	E	E	E	E	E	15

Table 8: Results of MCS for Forecasting RV of XMANA

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
HAR_log	3	-1.6491	5	-1.2971	11	1.3621	3	0.7465	2	-0.0235	1	-2.0024	4.17
HARJ_log	4	-1.3977	6	-1.2757	12	1.7081	2	-0.1349	3	0.1993	2	-1.5694	4.83
HARJ_sqrt	1	-2.6279	3	-1.4163	4	-1.4736	E	E	E	E	5	0.0598	7.17
HARQJ_log	10	0.6530	12	1.0017	13	1.7419	1	-0.9681	1	-0.1307	7	0.5030	7.33
HAR_sqrt	2	-2.0075	1	-1.4806	9	0.7983	E	E	E	E	4	-0.7041	7.67
CHAR_sqrt	6	-0.7842	7	-1.2243	5	-1.0276	E	E	E	E	6	0.3457	9
HAR	9	0.2101	2	-1.4341	1	-1.8647	E	E	E	E	E	E	9.5
CHAR_log	5	-0.9694	8	-1.1903	14	1.7878	E	E	E	E	3	-1.1574	10
CHAR	13	1.2111	4	-1.3210	2	-1.8094	E	E	E	E	E	E	10.67
CHARQ_sqrt	8	0.0605	10	-0.2102	6	-1.0134	E	E	E	E	E	E	11.5
CHARQ	12	1.0615	9	-0.7818	3	-1.6985	E	E	E	E	E	E	11.5
HARQJ_sqrt	7	-0.1269	14	1.1862	10	1.0749	E	E	E	E	9	1.5099	11.67
CHARQ_log	11	0.9276	13	1.0369	15	1.7943	E	E	E	E	8	0.9904	12.83
HARJ	14	1.2475	11	0.6090	8	-0.1241	E	E	E	E	E	E	13
HARQJ	E	E	15	1.4698	7	-0.4320	E	E	E	E	E	E	13.67

Table 9: Results of MCS for Forecasting RV of XMESY

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
CHAR_log	3	-1.2666	5	-1.1739	E	E	3	0.4149	1	-1.4886	1	-2.1257	4.67
HAR_log	4	-1.0229	4	-1.1845	E	E	E	E	4	0.6440	2	-1.6991	7.33
CHAR_sqrt	2	-1.4028	2	-1.2245	8	1.3731	E	E	E	E	3	-1.1093	7.5
HARJ_log	9	0.6918	15	1.3140	E	E	1	-0.9461	2	-0.1366	4	0.3282	7.67
CHARQ_log	11	0.8384	10	0.1669	E	E	2	0.1202	3	0.0314	5	0.4263	7.67
CHAR	5	-1.0139	1	-1.2632	4	-0.7364	E	E	E	E	7	0.9056	7.83
HARJ_sqrt	1	-1.5465	3	-1.2121	7	1.2683	E	E	E	E	9	1.7462	8.33
HARQJ_log	12	0.9903	13	1.1465	E	E	4	0.4410	5	1.3289	6	0.8800	9.17
CHARQ_sqrt	6	-0.8542	8	-0.8377	6	0.5295	E	E	E	E	8	1.5653	9.67
CHARQ	7	-0.4376	7	-1.1476	3	-1.2129	E	E	E	E	E	E	10.33
HAR	E	E	9	-0.1030	1	-1.8812	E	E	E	E	E	E	11.67
HARQJ	14	1.3294	11	1.0656	2	-1.3076	E	E	E	E	E	E	12
HAR_sqrt	8	-0.4193	6	-1.1651	E	E	E	E	E	E	E	E	12.33
HARJ	13	1.2068	12	1.0884	5	0.33093	E	E	E	E	E	E	12.5
HARQJ_sqrt	10	0.7942	14	1.2247	E	E	E	E	E	E	E	E	14

Table 10: Results of MCS for Forecasting RV of XTAST

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
CHAR_log	5	-1.1575	6	-1.1390	E	E	2	-0.3299	1	-0.8855	2	-1.7714	5.17
HAR_log	6	-1.0541	7	-1.1272	E	E	4	1.0318	3	-0.6088	1	-1.9212	6
HARJ_log	9	0.5690	10	1.1088	E	E	1	-1.7282	2	-0.6092	5	-0.1743	7
HAR_sqrt	2	-1.3012	5	-1.1428	4	-0.4906	E	E	E	E	4	-0.6007	7.5
CHAR_sqrt	3	-1.2941	2	-1.1497	8	1.3193	E	E	E	E	3	-0.7975	7.67
CHARQ_sqrt	4	-1.2792	3	-1.1497	7	1.3187	E	E	E	E	6	-0.0734	8.33
CHAR	7	-0.8573	1	-1.1500	3	-0.6446	E	E	E	E	10	1.3059	8.5
HARQJ_log	11	1.0115	13	1.1347	E	E	3	-0.2535	4	1.0973	9	0.6717	9.17
CHARQ_log	12	1.0130	12	1.1209	E	E	5	1.2446	5	1.2722	7	0.4421	9.33
HARJ_sqrt	1	-1.5518	4	-1.1481	E	E	E	E	E	E	8	0.5182	9.67
CHARQ	8	-0.7407	8	-1.1235	5	0.4105	E	E	E	E	E	E	11
HARQJ_sqrt	10	0.8191	11	1.1178	6	0.6200	E	E	E	E	11	1.3895	11.33
HAR	E	E	9	-1.1171	1	-1.2117	E	E	E	E	E	E	11.67
HARQJ	13	1.2511	14	1.1440	2	-0.9955	E	E	E	E	E	E	12.33
HARJ	14	1.3168	15	1.1488	E	E	E	E	E	E	E	E	14.83

Table 11: Results of MCS for Forecasting RV of XTEKS

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
CHAR_log	4	-1.0351	6	-1.1298	E	E	3	0.3914	2	-0.1787	1	-1.4443	5.17
HAR_log	5	-0.9166	8	-1.1038	12	1.6895	2	0.0303	3	-0.1672	2	-1.4199	5.33
HARJ_log	8	0.4728	10	1.0266	E	E	1	-1.0743	1	-0.5984	6	0.0154	6.83
HARJ_sqrt	1	-1.4884	3	-1.1734	5	-1.4190	E	E	E	E	3	-1.0935	7
CHAR_sqrt	2	-1.1805	4	-1.1498	7	-0.2932	E	E	E	E	4	-0.9698	7.83
HARQJ_log	10	0.9733	12	1.1099	11	1.6765	4	0.6124	4	0.6493	7	0.5613	8
HAR_sqrt	3	-1.1732	5	-1.1397	10	1.2547	E	E	E	E	5	-0.7224	8.83
CHAR	6	-0.7313	7	-1.1216	4	-1.5045	E	E	E	E	8	0.6220	9.17
CHARQ_sqrt	7	0.4239	1	-1.1994	8	0.6996	E	E	E	E	E	E	10.17
HAR	E	E	2	-1.1814	1	-3.5012	E	E	E	E	E	E	10.5
HARQJ_sqrt	9	0.9115	11	1.1069	9	1.1526	E	E	E	E	10	1.1545	11.5
CHARQ	E	E	9	-1.0278	3	-1.6109	E	E	E	E	E	E	12
HARQJ	12	1.3366	14	1.1417	2	-2.6964	E	E	E	E	E	E	12.17
CHARQ_log	11	1.1337	13	1.1253	E	E	E	E	E	E	9	0.9667	13
HARJ	E	E	15	1.2064	6	-1.0591	E	E	E	E	E	E	13.5

Table 12: Results of MCS for Forecasting RV of XUMAL

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
CHAR_log	4	-1.0853	7	-1.0031	E	E	3	-0.0702	1	-1.1574	1	-2.0966	5.17
HAR_log	6	-0.9250	6	-1.0060	11	1.4696	5	1.0868	2	-0.0639	2	-1.8771	5.33
HARJ_log	9	0.6893	12	1.0383	13	1.6359	1	-1.6746	3	-0.0104	5	-0.1580	7.17
CHAR_sqrt	2	-1.2172	2	-1.0381	8	0.5323	E	E	E	E	3	-1.0811	7.5
CHAR	5	-0.9883	1	-1.0726	2	-1.7806	E	E	E	E	8	0.6847	7.67
CHARQ_sqrt	1	-1.2493	4	-1.0179	10	0.9988	E	E	E	E	4	-0.7640	8.17
HARQJ_log	12	0.9420	11	1.0241	12	1.5634	2	-0.5191	5	0.7875	7	0.4992	8.17
HARJ_sqrt	3	-1.1663	3	-1.0298	6	-0.1396	E	E	E	E	9	0.8086	8.5
CHARQ_log	11	0.9323	13	1.0419	E	E	4	1.0740	4	0.7507	6	0.4074	8.83
CHARQ	7	-0.7233	5	-1.0077	3	-1.2874	E	E	E	E	11	1.1063	9.33
HAR_sqrt	8	-0.5974	8	-0.9787	7	0.4267	E	E	E	E	12	1.1545	10.83
HARQJ_sqrt	10	0.8506	10	0.9882	9	0.5335	E	E	E	E	10	1.0251	11.5
HARQJ	14	1.2865	9	0.9840	4	-1.1695	E	E	E	E	E	E	12
HAR	E	E	E	E	1	-2.5432	E	E	E	E	E	E	12.67
HARJ	13	1.2760	14	1.0504	5	-0.5699	E	E	E	E	E	E	12.83

Table 13: Results of MCS for Forecasting RV of XUSIN

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
HARJ_log	8	-0.4054	1	-1.1219	E	E	1	-0.5489	2	-0.3499	4	-0.3287	5.17
CHAR_log	5	-1.0657	8	-1.0178	E	E	2	-0.4971	1	-1.0771	1	-1.5316	5.33
HAR_log	6	-1.0522	4	-1.0292	11	1.4659	E	E	3	0.1644	2	-1.4252	6.83
CHAR	7	-0.9076	2	-1.0346	1	-1.5646	E	E	E	E	6	0.2112	7.67
HARJ_sqrt	2	-1.2181	6	-1.0276	5	-0.4750	E	E	E	E	5	-0.2696	8
CHAR_sqrt	3	-1.0989	7	-1.0223	6	-0.1480	E	E	E	E	3	-1.0107	8.17
CHARQ_sqrt	1	-2.2076	5	-1.0286	4	-0.4791	E	E	E	E	10	1.0482	8.33
HAR_sqrt	4	-1.0911	3	-1.0306	9	1.0846	E	E	E	E	7	0.2665	8.83
HARQJ_log	10	0.5516	11	0.9762	E	E	4	1.1081	5	1.1301	8	0.4908	8.83
CHARQ_log	11	0.8315	14	1.0185	E	E	3	0.0946	4	0.4000	9	0.6196	9
CHARQ	9	-0.0044	9	-0.9716	3	-1.2215	E	E	E	E	E	E	11
HARJ	13	0.9411	12	0.9976	2	-1.5554	E	E	E	E	E	E	12
HARQJ_sqrt	12	0.9362	13	1.0171	10	1.3301	E	E	E	E	11	1.4451	12.67
HAR	14	1.2743	10	0.9564	8	0.3832	E	E	E	E	E	E	12.83
HARQJ	15	1.4373	15	1.0481	7	0.1278	E	E	E	E	E	E	14

Table 14: Results of MCS for Forecasting RV of XU030

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
CHAR_log	4	-1.1909	6	-0.9911	E	E	2	-0.1460	1	-1.1251	1	-1.9727	4.83
HARJ_log	9	0.0293	9	1.0006	12	1.6237	1	-1.1038	2	-0.7176	3	-0.8135	6
HAR_log	5	-1.0187	4	-1.0271	11	1.4830	E	E	3	0.2315	2	-1.8179	6.67
HARJ_sqrt	1	-1.6597	2	-1.1033	4	-1.5190	E	E	E	E	9	1.2863	7.67
CHAR_sqrt	3	-1.3750	3	-1.0539	7	0.3498	E	E	E	E	4	-0.6929	7.83
CHAR	6	-0.9436	1	-1.1034	2	-2.1568	E	E	E	E	10	1.4207	8.17
CHARQ_log	11	0.9678	13	1.0945	E	E	4	1.3111	5	0.8788	5	0.4172	8.83
HARQJ_log	12	1.0898	E	E	13	1.7073	3	0.1154	4	0.8222	6	0.4513	8.83
CHARQ_sqrt	2	-1.5703	8	-0.9413	8	0.3663	E	E	E	E	7	0.9006	9.17
CHARQ	7	-0.6831	5	-1.0082	3	-1.5271	E	E	E	E	E	E	10
HAR_sqrt	8	-0.4575	7	-0.9604	9	0.5625	E	E	E	E	E	E	11.5
HARQJ_sqrt	10	0.9639	11	1.0177	10	0.9227	E	E	E	E	8	1.1723	11.5
HARQJ	14	1.5371	10	1.0088	5	-0.8923	E	E	E	E	E	E	12.33
HARJ	13	1.1579	12	1.0201	6	-0.4235	E	E	E	E	E	E	12.67
HAR	E	E	E	E	1	-3.0114	E	E	E	E	E	E	12.67

Table 15: Results of MCS for Forecasting RV of XU100

	SE1		SE2		QLIKE		R2LOG		AE1		AE2		
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	AverageRanking
CHAR_log	4	-1.0675	5	-0.9811	E	E	2	-0.2277	2	-1.0373	1	-1.8112	4.83
HARJ_log	9	0.0062	9	0.9263	E	E	1	-1.2069	1	-1.1391	3	-0.8652	6.33
HAR_log	5	-0.9979	4	-1.0123	11	1.5327	E	E	3	-0.0199	2	-1.8044	6.67
CHAR_sqrt	2	-1.2145	3	-1.0176	7	0.8277	E	E	E	E	4	-0.7981	7.67
HARJ_sqrt	1	-1.5515	2	-1.0552	6	-0.9388	E	E	E	E	8	0.8429	7.83
CHAR	6	-0.8985	1	-1.0601	2	-1.8200	E	E	E	E	9	0.9069	8
CHARQ_log	12	0.9216	13	1.0328	E	E	4	1.3267	4	0.8532	5	0.5103	8.83
CHARQ_sqrt	3	-1.1456	8	-0.9301	8	0.9546	E	E	E	E	7	0.8317	9.33
HARQJ_log	13	1.1394	E	E	E	E	3	0.3666	5	1.2377	6	0.6974	9.5
HAR_sqrt	7	-0.7076	6	-0.9772	9	0.9935	E	E	E	E	11	1.3593	10.5
CHARQ	8	-0.4055	7	-0.9370	5	-1.2293	E	E	E	E	E	E	10.83
HARJ	11	0.9081	10	0.9694	3	-1.5810	E	E	E	E	E	E	11.5
HARQJ_sqrt	10	0.8886	12	1.0071	10	1.4361	E	E	E	E	10	1.0975	12
HARQJ	14	1.4030	11	0.9999	4	-1.4220	E	E	E	E	E	E	12.33
HAR	E	E	E	E	1	-2.9442	E	E	E	E	E	E	12.67

FIGURES

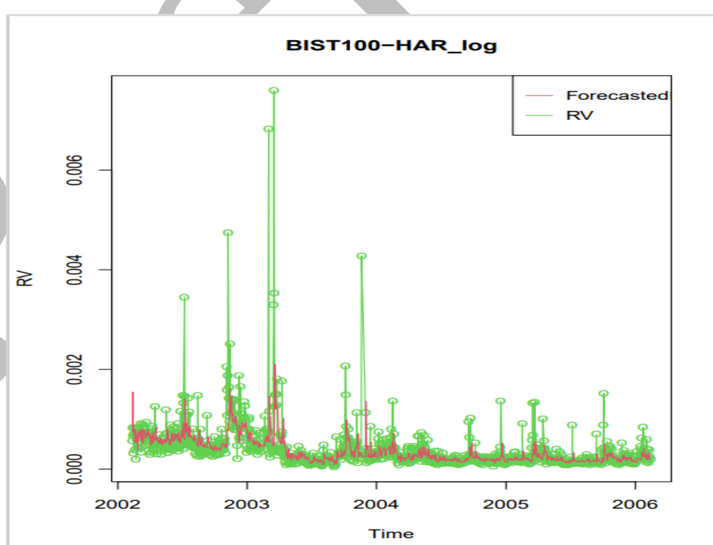

Figure 1:Forecasting Results of Plain HAR-RV on Log Series (2002-2006)

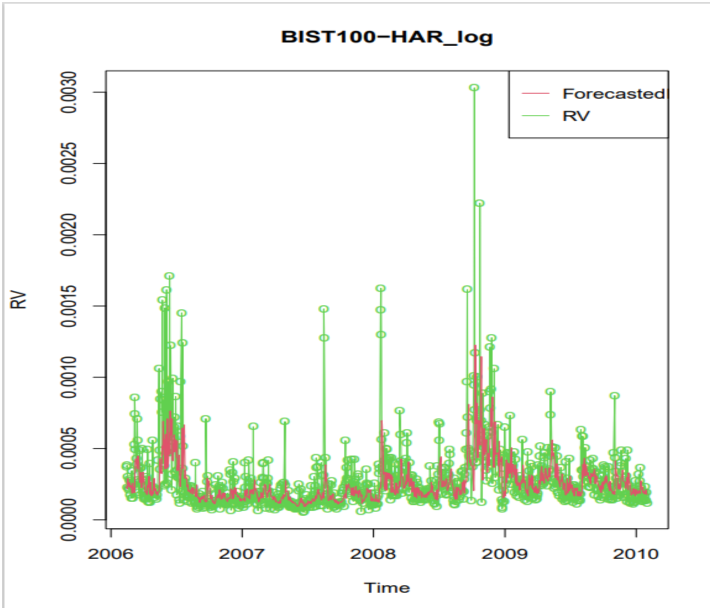

Figure 2:Forecasting Results of Plain HAR-RV on Log Series (2006-2010)

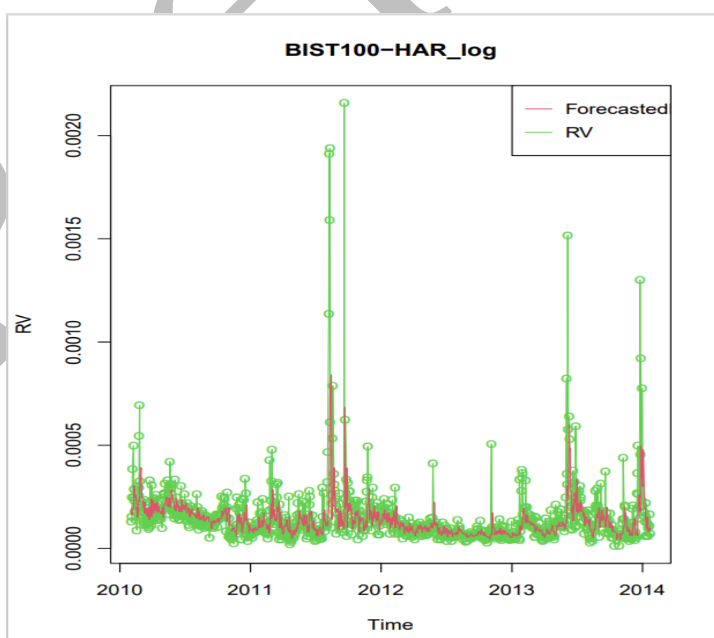

Figure 3:Forecasting Results of Plain HAR-RV on Log Series (2010-2014)

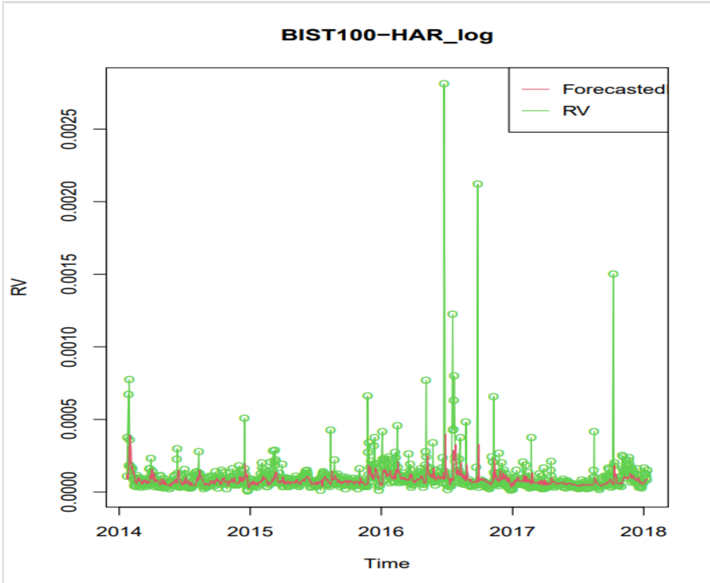

Figure 4:Forecasting Results of Plain HAR-RV on Log Series (2014-2018)

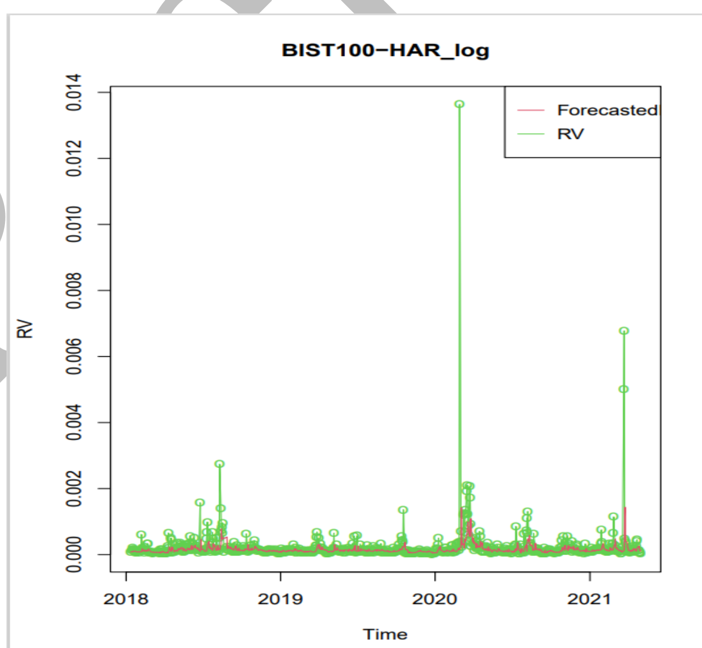

Figure 5:Forecasting Results of Plain HAR-RV on Log Series (2018-2021)